OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-2 : LIMITS, CONTINUITY AND DIFFERENTIABILITY

UNIT TEST-1

1. The value of the limit $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{4\sqrt{2}(\sin 3x+\sin x)}{\left(2\sin 2x\sin\frac{3x}{2}+\cos\frac{5x}{2}\right)-\left(\sqrt{2}+\sqrt{2}\cos 2x+\cos\frac{3x}{2}\right)}$ is _____

2. Let k and m be positive real numbers such that the function $f(x)=\begin{cases}3x^2+k\sqrt{x+1}, & 0<x<1\\ mx^2+k^2, & x\ge 1\end{cases}$ is differentiable for all $x>0$. Then $\dfrac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to ________.

3. Let $f:(-2,\ 2)\to R$ be defined by $f(x)=\begin{cases}x[x], & -2<x<0\\ (x-1)[x], & 0\le x<2\end{cases}$

Where $[x]$ denotes the greatest integer function. If m *and* n respectively are the number of points in $(-2, 2)$ at which $y=|f(x)|$ is not continuous and not differentiable, then $m+n$ is equal to _________.

Hints and Solutions

1. (8)

$$\lim_{x\to\frac{\pi}{2}} \frac{4\sqrt{2}\cdot 2\sin 2x\cos x}{2\sin 2x\sin\frac{3x}{2}+\left(\cos\frac{5x}{2}-\cos\frac{3x}{2}\right)-\sqrt{2}(1+\cos 2x)}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{8\sqrt{2}\cdot 2\sin x\cos x\cos x}{2\sin 2x\sin\frac{3x}{2}-2\sin 2x\sin\frac{x}{2}-2\sqrt{2}\cos^2 x}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{2\sin 2x\left(\sin\frac{3x}{2}-\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{4\sin x\cos x\left(2\cos x\cdot\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$=\frac{16\sqrt{2}\sin x\cos^2 x}{2\cos^2 x\left(4\sin x\sin\frac{x}{2}-\sqrt{2}\right)}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{8\sqrt{2}\sin x}{4\sin x\cdot\sin\frac{x}{2}-\sqrt{2}}=8$$

2. (309) For continuity at $x=1$

$f(1^-)=f(1^+)$

$\Rightarrow 3+k\sqrt{2}=m+k^2$ ...(i)

Now, $f'(x)\Big|_{x=1^-}=6x+\dfrac{k}{2\sqrt{x+1}}\Big|_{x+1^-}=6+\dfrac{k}{2\sqrt{2}}$

Also, $f'(x)\Big|_{x=1^+}=2mn\Big|_{x=1^+}=2m$

For differentiability at $x=1$

$6+\dfrac{k}{2\sqrt{2}}=2m$ ...(ii)

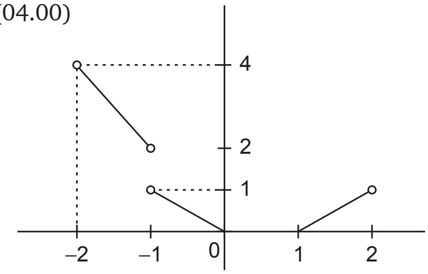
From (i) and (ii), $k=\dfrac{7}{4\sqrt{2}}$ and $m=\dfrac{103}{32}$

Now, $f'(8) = 2mn\big|_{x=8} = 16m = \frac{103}{2}$

And, $f'\left(\frac{1}{8}\right) = \left(6x + \frac{k}{2\sqrt{x+1}}\right)\Bigg|_{x=\frac{1}{8}} = \frac{4}{3}$

Now, $\frac{8.f'(8)}{f'\left(\frac{1}{8}\right)} = 309$

3. (04.00)



$y = f(x)$ is same as $y = |f(x)|$

$m = 1, n = 3$